Prime numbers and fundamental theorem of arithmetic

Fundamental theorem of arithmetic

1) Is the number a is divisible by b?
   a) $a = 2^5 \cdot 3^4 \cdot 5^2 \cdot 7^3$, $b = 216$
   b) $a = 2^3 \cdot 3^5 \cdot 11^2 \cdot 17$, $b = 144$

   Source: Projekt MmF

2) How many zeros are at the end of the number?
   a) $1 \cdot 2 \cdot 3 \cdot \ldots \cdot 15 \cdot 16$
   b) $1 \cdot 2 \cdot 3 \cdot \ldots \cdot 25 \cdot 26$

   Source: [4], #64

3) Is there an integer whose product of digits is a) 2000; b) 2016; c) 2022?

   Source: [3], #88254

4) Which of the following numbers are not dividers of the number $18^{2017} + 18^{2018}$: 8, 18, 28, 38, 48?

   Source: Kangaroo

5) The product of two numbers, which are not divisible by 10, is 1000. Find these numbers.

   Source: [3], #115462

6) What smallest number of numbers from the sequence $10, 20, 30, \ldots, 90$ can be removed so that the product of the remaining numbers is a square?

   Source: Kangaroo 2019, grade 9-10, problem 28

7) Alice made the set of prime numbers less than 100 using the digits 1, 2, 3, 4, and 5 exactly once and using no other digits. What number must be in Alice's set?

   Source: [1], 2018, grade 7-8, #20

Prime numbers and factorising of polynomials

8) Numbers a, b are squares of natural numbers. The number a – b is prime. Which of proposed in the answers can be equal to b?

   a) 100; b) 144; c) 256; d) 900; e) 100000.

   Source: [1], 2021, grade 9-10, #26

9) Find all the natural numbers n, such that polynom
   a) $n^2 - 1$
   b) $n^2 + n - 2$
   c) $n^2 + 6n - 16$

   takes prime (positive) value.

   Source: Projekt MmF

10) Is it true that for every natural n the polynom $n^2 + n + 41$ takes only prime values?

    Source: [3], #60473

11) Show that numbers
    a) 9919
    b) 1081
    c) 999 999 973

    are not prime.

    Source: [4], 12.25

Prime numbers and the theory of residuals

12) Find all the sequences n, $n + 2$, $n + 4$, where all the numbers are prime.

    Source: [3], #60469

13) Numbers p and $8p^2 + 1$ are prime. Find p.

    Source: [4], #12.22

14) Find all natural numbers n, such that both numbers $2^n - 1$ and $2^n + 1$ are prime.

    Source: [4], #12.32

15) Find all numbers n such that the number $n^4 + 4$ is prime.

    Source: [4], #12.33

Bibliography

This problem set has been composed by the team of Projekt MmF. The sources of all problems are given. The translations and slight adaptations are due to us.

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