Level 5 Module 1
exemplar with comments

2015-DSE
MATH EP
M1

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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2015

Candidate Number									

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

7. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 1 E 0 0 1 *

SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where a and b are constants:

x	2	3	5	7	9
$P(X = x)$	0.08	0.15	a	0.45	b

It is given that $E(X) = 5.64$. Find

(a) a and b,

(b) $E((6-5X)^2)$ and $\text{Var}(6-5X)$.

(6 marks)

a) $0.08 + 0.15 + a + 0.45 + b = 1$

$a + b = 0.32$ -①

$2(0.08) + 3(0.15) + 5a + 7(0.45) + 9b = 5.64$

$5a + 9b = 1.88$ -②

from ①,

$a = 0.32 - b$ -③

sub ③ into ②

$5(0.32 - b) + 9b = 1.88$

$1.6 - 5b + 9b = 1.88$

$b = 0.07$

$a = 0.25$

b) $E(6-5X)^2)$

$= 36(0.08) + 81(0.15) + 361(0.25) + 841(0.45) + 1521(0.07)$

$= 588.6$

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$= 35.64 - 5.64^2$

$= 3.8304$

$\text{Var}(6-5X) = 5^2(3.8304)$

$= 95.76$

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2. A and B are two events. Suppose that $P(A) = 0.3$, $P(B) = 0.28$ and $P(B' \mid A') = 0.6$, where A' and B' are the complementary events of A and B respectively.

(a) Find $P(A' \cap B')$ and $P(A' \cap B)$.

(b) Are A and B mutually exclusive? Explain your answer.

(6 marks)

a) $P(B' \mid A') = \frac{P(A' \cap B')}{P(A')}$

$0.6 = \frac{P(A' \cap B')}{0.7}$

$P(A' \cap B') = 0.42$

$P(A' \cap B) = 1 - P(A' \cap B') - P(A)$

$P(A' \cap B) = 1 - 0.42 - 0.3$

$P(A' \cap B) = 0.28$

b) $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

$P(A' \cup B') = 0.7 + 0.72 - 0.42$

$P(A' \cup B') = 1$

$P(A' \cup B') = 1 - P(A \cap B)$

$1 = 1 - P(A \cap B)$

$P(A \cap B) = 0$

∴ Yes, A and B are mutually exclusive.

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3. A bag contains 2 white balls and 5 yellow balls. In a survey, each interviewee draws a ball randomly from the bag. If a white ball is drawn, then the interviewee considers the question 'Are you a smoker?'. If a yellow ball is drawn, then the interviewee considers the question 'Are you a non-smoker?'. Finally, the interviewee answers either 'Yes' or 'No'. Let p be the probability that a randomly selected interviewee is a smoker.

(a) Express, in terms of p, the probability that a randomly selected interviewee answers 'Yes'.

(b) In this survey, 50 out of 91 interviewees answer 'Yes'.

(i) Find p.

(ii) Given that an interviewee answers 'No', find the probability that the interviewee is a non-smoker.

(6 marks)

3a) P(interviewee answers 'Yes')

$= \frac{2}{7} \times p + \frac{5}{7}(1-p)$

$= \frac{2}{7}p + \frac{5}{7} - \frac{5}{7}p$

$= \frac{5}{7} - \frac{3}{7}p$

b) $\frac{5}{7} - \frac{3}{7}p = \frac{50}{91}$

$p = \frac{5}{13}$

ii) The required probability

$= \frac{\frac{2}{7} \times \frac{8}{13}}{\frac{2}{7} \times \frac{8}{13} + \frac{5}{7} \times \frac{5}{13}}$

$= \frac{\frac{16}{91}}{\frac{41}{91}}$

$= \frac{16}{41}$

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4. A manufacturer of brand B biscuits starts a promotion plan by giving one reward points card in each packet of biscuits. It is found that 75% of the packets of brand B biscuits contain 3-point cards and the rest contain 7-point cards. A total of 20 points or more can be exchanged for a gift coupon. John buys 4 packets of brand B biscuits and he opens them one by one.

(a) Find the probability that John gets the first 7-point card when the 4th packet of brand B biscuits has been opened.

(b) Find the probability that John can exchange for a gift coupon.

(c) Given that John can exchange for a gift coupon, find the probability that he gets a 7-point card when the 4th packet of brand B biscuits has been opened.

(7 marks)

Answers written in the margins will not be marked.

a) The required probability

$= C_0^3 (0.25)^0 (0.75)^3 (0.25)$

$= 0.1055$ (corr. to 4 dps)

b) The required probability

$= C_2^4 (0.25)^2 (0.75)^2 + C_3^4 (0.75)(0.25)^3 + C_4^4 (0.25)^4$

$= 0.2617$ (corr. to 4 dps)

c) The required probability

$= \dfrac{C_1^3 (0.25)(0.75)^2 (0.25) + C_2^3 (0.25)^2 (0.75)(0.25) + C_4^4 (0.25)^4}{C_2^4 (0.25)^2 (0.75)^2 + C_3^4 (0.75)(0.25)^3 + C_4^4 (0.25)^4}$

$= \dfrac{0.14453125}{0.2617187 5}$

$= 0.5522$ (corr. to 4 dps)

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5. (a) Expand e^{-4x} in ascending powers of x as far as the term in x^2.

(b) Find the coefficient of x^2 in the expansion of $\frac{(2+x)^5}{e^{4x}}$.

(5 marks)

a) e^{-4x}

$= 1 - 4x + \frac{(4x)^2}{2} + \cdots$

$= 1 - 4x + 8x^2 + \cdots$

b) $(2+x)^5$

$= C_0^5 (2)^5 (x)^0 + C_1^5 (2)^4 (x) + C_2^5 (2)^3 (x)^2 + \cdots$

$= 32 + 80x + 80x^2 + \cdots$

$\frac{(2+x)^5}{e^{4x}}$

$= (2+x)^5 e^{-4x}$

$= (32 + 80x + 80x^2 + \cdots)(1 - 4x + 8x^2 + \cdots)$

$= 32 - 128x + 256x^2 + 80x - 320x^2 + 80x^2 + \cdots$

$= 32 - 48x + 16x^2 + \cdots$

$\therefore$ The coefficient of $x^2 = 16$

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6. Consider the curves $C_1 : y = e^{2x} + e^4$ and $C_2 : y = e^{x+3} + e^{x+1}$.

(a) Find the x-coordinates of the two points of intersection of C_1 and C_2.

(b) Express, in terms of e, the area of the region bounded by C_1 and C_2.

(6 marks)

a) $\begin{cases} y = e^{2x} + e^4 & -(1) \\ y = e^{x+3} + e^{x+1} & -(2) \end{cases}$

sub (1) into (2)

$e^{2x} + e^4 = e^{x+3} + e^{x+1}$

$e^{2x} + e^4 = e^x \cdot e^3 + e^x \cdot e^1$

$e^{2x} + e^4 = e^x (e^3 + e^1)$

$e^{2x} - e^x (e^3 + e) + e^4 = 0$

$e^x = 20.085537$ or $e^x = 2.718281828$

$x = 3$ or $x = 1$

b) The area of the region

$= \int_1^3 -[(e^{2x} + e^4) + (e^{x+3} + e^{x+1})]\,dx$

$= \int_1^3 -(e^{2x} + e^4 + e^{x+3} + e^{x+1})\,dx$

$= \left[-\frac{e^{2x}}{2} - e^4 x + e^{x+3} + e^{x+1}\right]_1^3$

$= (-\frac{e^6}{2} - 3e^4 + e^6 + e^4) - (-\frac{e^2}{2} - e^4 + e^4 + e^2)$

$= \frac{1}{2}e^6 - 2e^4 - (\frac{1}{2}e^2)$

$= \frac{1}{2}e^6 - 2e^4 - \frac{1}{2}e^2$ //

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7. Consider the curve $C : y = x\sqrt{2x^2+1}$.

(a) Find $\frac{dy}{dx}$.

(b) Two of the tangents to C are perpendicular to the straight line $3x+17y=0$. Find the equations of the two tangents.

(7 marks)

a) $y = x\sqrt{2x^2+1}$

$\frac{dy}{dx} = x\left(\frac{1}{2}\right)(2x^2+1)^{-\frac{1}{2}}(4x) + (2x^2+1)^{\frac{1}{2}}(1)$

$= (2x^2+1)^{-\frac{1}{2}}(2x^2 + 2x^2 + 1)$

$= \frac{4x^2+1}{\sqrt{2x^2+1}}$

b) slope of straight line $= \frac{-3}{17}$

slope of tangent $= \frac{17}{3}$

$\frac{4x^2+1}{\sqrt{2x^2+1}} = \frac{17}{3}$

$\frac{12x^2+3}{17} = \sqrt{2x^2+1}$

$\frac{144x^4+72x^2+9}{289} = 2x^2+1$

$144x^4 + 72x^2 + 9 = 578x^2 + 289$

$144x^4 - 506x^2 - 280 = 0$

$x^2 = 4$ or $x^2 = -\frac{35}{72}$ (rejected)

$x = 2$ or -2

when $x = 2$, $y = 6$

when $x = -2$, $y = -6$

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① equation of tangent

$y - 6 = \frac{17}{3}(x - 2)$

$3y - 18 = 17x - 34$

$17x - 3y - 16 = 0$

② equation of tangent

$y + 6 = \frac{17}{3}(x + 2)$

$3y + 18 = 17x + 34$

$17x - 3y + 16 = 0$

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8. (a) Express $\frac{d}{dx}\left((x^6+1)\ln(x^2+1)\right)$ in the form $f(x)+g(x)\ln(x^2+1)$, where $f(x)$ and $g(x)$ are polynomials.

(b) Find $\int x^5 \ln(x^2+1)\,dx$.

(7 marks)

a) $\frac{d}{dx}\left((x^6+1)(\ln(x^2+1))\right)$

$= (x^6+1)\left(\frac{2x}{x^2+1}\right) + \ln(x^2+1)(6x^5)$

$= \frac{2x(x^6+1)}{x^2+1} + 6x^5\ln(x^2+1)$

b) $\int\left[\frac{2x(x^6+1)}{x^2+1} + 6x^5\ln(x^2+1)\right]dx = (x^6+1)\ln(x^2+1)+C$

$\int \frac{2x(x^6+1)}{x^2+1}\,dx + 6\int x^5\ln(x^2+1)\,dx = (x^6+1)\ln(x^2+1)+C$

$6\int x^5\ln(x^2+1)\,dx = (x^6+1)\ln(x^2+1) - \int \frac{2x(x^6+1)}{x^2+1}\,dx + C$

$\int \frac{2x(x^6+1)}{x^2+1}\,dx$

Let $u = x^2+1$ $\quad x^2 = u-1$

$\frac{du}{dx} = 2x$

$\int \frac{(x^6+1)}{u}\,du$

$= \int \frac{(u-1)^3+1}{u}\,du$

$= \int \frac{u^3-3u^2+3u-1+1}{u}\,du$

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$= \int (u^3 + 3u^2 + 3u)\, u^{-1}\, du$

$= \int (u^2 + 3u + 3)\, du$

$= \frac{u^3}{3} + \frac{3u^2}{2} + 3u + C$

$= \frac{(x^2+1)}{3} + \frac{3(x^2+1)^2}{2} + 3(x^2+1) + C$

$6\int x^5 \ln(x^2+1)\, dx = (x^6+1)\ln(x^2+1) - \frac{(x^2+1)}{3} - \frac{3(x^2+1)^2}{2} + 3(x^2+1) + C$

$\int x^5 \ln(x^2+1)\, dx = \frac{(x^6+1)(\ln(x^2+1)}{6} - \frac{(x^2+1)}{18} - \frac{(x^2+1)^2}{4} + \frac{(x^2+1)}{2} + C$

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SECTION B (50 marks)

9. The speeds of cars passing a checkpoint on a highway follow a normal distribution with a mean of μ km/h and a standard deviation of 16 km/h.

(a) A survey on the speeds of cars to estimate μ is conducted.

(i) A random sample of 25 cars is taken and the stem-and-leaf diagram below shows the distribution of their speeds (in km/h) :

Stem (tens)	Leaf (units)
6	0 0 1 1 1 2 2 3 4 4 5 5 6 6 7
7	1 1 2 3 5 5 6
8	3 6 7

Find a 95% confidence interval for μ .

(ii) Find the least sample size to be taken such that the width of a 97.5% confidence interval for μ is less than 9 .

(7 marks)

(b) Suppose that $\mu = 66$. If the speed of a car passing the checkpoint exceeds 90 km/h , a penalty ticket will be issued.

(i) If a car passes the checkpoint, find the probability that a penalty ticket will be issued.

(ii) If 12 cars pass the checkpoint, find the probability that more than 2 penalty tickets will be issued.

(5 marks)

a7) $\mu = \frac{1716}{25}$

$= 68.64$

Let x km/h be the speeds of cars in 25 sample.

$x \sim N(68.64, \frac{16^2}{25})$ approximately

95% confidence interval

$= (68.64 - 1.96\sqrt{\frac{16^2}{25}},\ 68.64 + 1.96\sqrt{\frac{16^2}{25}})$

$= (62.368,\ 74.912)$

Let n be the sample size

77) $(68.64 + 2.24(\frac{16}{\sqrt{n}})) - (68.64 - 2.24(\frac{16}{\sqrt{n}})) < 9$

$2[2.24(\frac{16}{\sqrt{n}})] < 9$

$n > 63.4324$ (corr. to 4 dps)

∴ The least sample size is 64.

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b7) $P(X > 90)$

$= P\left(Z > \frac{90-66}{16}\right)$

$= P(Z > 1.5)$

$= 0.5 - P(0 < z < 1.5)$

$= 0.5 - 0.4332$

$= 0.0668$

$\therefore$ The required probability $= 0.0668$.

7) The required probability

$= 1 - [C_0^{12}(0.0668)^0(0.9332)^{12} + C_1^{12}(0.0668)(0.9332)^{11}$

$+ C_2^{12}(0.0668)^2(0.9332)^{10}]$

$= 0.0416$ (corr. to 4 d.p.)

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10. The number of customers buying tickets at cinema A in a minute can be modelled by a Poisson distribution with a mean of 3.2. The probability distribution of the number of tickets bought by a customer at cinema A is shown in the following table:

Number of tickets bought	1	2	3	4	5	6	≥ 7
Probability	0.12	0.7	0.08	0.04	0.03	0.02	0.01

(a) Find the probability that fewer than 4 customers buy tickets at cinema A in a certain minute. (3 marks)

(b) Find the probability that the 8th customer buying tickets at cinema A is the 3rd customer who buys 2 tickets. (2 marks)

(c) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and each of them buys 2 tickets. (2 marks)

(d) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and they buy a total of 6 tickets. (3 marks)

(e) Given that fewer than 4 customers buy tickets at cinema A in a certain minute, find the probability that they buy a total of 6 tickets. (3 marks)

10a) Let X be the number of customers buying tickets at cinema A follows Poisson distribution.

$X \sim Po(3.2)$

$P(X < 4)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= e^{-3.2} + e^{-3.2}(3.2) + \frac{e^{-3.2}(3.2)^2}{2} + \frac{e^{-3.2}(3.2)^3}{6}$

$= 0.6025$ (corr. to 4 d.p.)

b) The required probability

$= C_2^7 (0.7)^2 (0.3)^5 (0.7)$

$= 0.0175$ (corr. to 4 d.p.)

c) The required probability

$= \frac{e^{-3.2} 3.2^3}{6} (0.7^3)$

$= 0.0764$ (corr. to 4 d.p.)

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(d) The required probability

$$= \frac{e^{-3.2}3.2^3}{6}\left[3\times(0.12)(0.7)(0.08)+3(0.12)^2(0.04)+0.7^3\right]$$

$$= \frac{e^{-3.2}3.2^3}{6}(0.364888)$$

$$= 0.0812 \text{ (corr. to 4 dp)}$$

(e) The required probability

$$= \frac{e^{-3.2}(3.2)(0.08)+\frac{e^{-3.2}3.2^2}{2}[2(0.12)(0.03)+2(0.7)(0.04)+0.28^3]}{e^{-3.2}(3.2)+\frac{e^{-3.2}(3.2^2)}{2}+\frac{e^{-3.2}(3.2)^3}{6}} + 0.0812$$

$$= \frac{0.097135532}{0.56175752}$$

$$= 0.1729 \text{ (corr. to 4 d.p.)}$$

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11. An engineer models the rates of change of the amount of oil produced (in hundred barrels per day) by oil companies X and Y respectively by

$$f(t) = \ln(e^t - t) \text{ and } g(t) = \frac{8t}{1+t},$$

where $t\,(2 \le t \le 12)$ is the time measured in days.

(a) Using the trapezoidal rule with 5 sub-intervals, estimate the total amount of oil produced by oil company X from $t = 2$ to $t = 12$. (3 marks)

(b) Determine whether the estimate in (a) is an over-estimate or an under-estimate. Explain your answer. (3 marks)

(c) Find $\int \frac{t}{1+t}\,dt$. (3 marks)

(d) The engineer claims that the total amount of oil produced by oil company X from $t = 2$ to $t = 12$ is less than that of oil company Y. Do you agree? Explain your answer. (3 marks)

a) $\Delta x = \frac{12-2}{5}$

$= 2$

$\int_2^{12} \ln(e^t - t)\,dt \approx \frac{2}{2}[1.68437049 + 2(3.92391.50) + 2(5.98501.6) + 2(7.99731.27) + 2(9.99954590) + 11.9999 \times 6 \times 7]$

≈ 69.4959 (corr to 4 d.p.)

b) $f'(t) = \frac{1}{e^t - t}(e^t - 1)$

$= \frac{e^t - 1}{e^t - t}$

$f''(t) = \frac{(e^t - t)(e^t) - (e^t - 1)(e^t - 1)}{(e^t - t)^2}$

$= \frac{(e^t - t)e^t - (e^t - 1)^2}{(e^t - t)^2}$

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$$f''(t) = \frac{-(te^t - 2e^t + 1)}{(e^t - t)^2}$$

$\because (e^t - t)^2 > 0$ and $(te^t - 2e^t + 1) > 0$ for $2 \le t \le 12$

$\therefore f''(t) < 0$

$\therefore$ underestimate.

(c) $\int \frac{t}{1+t}\, dt$

Let $u = 1 + t$

$\frac{du}{dt} = 1$

$t = u - 1$

$\int (u-1)u^{-1}\, du$

$= \int 1 - \frac{1}{u}\, du$

$= (1+t) - \ln(1+t) + C$

(d) $\int_2^{12} \frac{8t}{1+t}\, dt$

$= 8\int_2^{12} \frac{t}{1+t}\, dt$

$= 8\left[(1+t) - \ln(1+t)\right]_2^{12}$

$= 8\left[13 - \ln 13 - (3 - \ln 3)\right]$

$= 68.2693$ (corr. to 4 dp)

< 69.495

$\therefore$ No, the claim is incorrect.

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12. In an experiment, the temperature (in °C) of a certain liquid can be modelled by

$$S = \frac{200}{1 + a2^{bt}},$$

where a and b are constants and t is the number of hours elapsed since the start of the experiment.

(a) Express $\ln\left(\frac{200}{S} - 1\right)$ as a linear function of t. (2 marks)

(b) It is found that the intercepts on the vertical axis and the horizontal axis of the graph of the linear function obtained in (a) are $\ln 4$ and 4 respectively.

(i) Find a and b.

(ii) Find $\frac{dS}{dt}$ and $\frac{d^2S}{dt^2}$.

(iii) Describe how S and $\frac{dS}{dt}$ vary during the first 48 hours after the start of the experiment. Explain your answer.

(11 marks)

a) $S = \frac{200}{1 + a2^{bt}}$

$\frac{200}{S} = 1 + a2^{bt}$

$\frac{200}{S} - 1 = a2^{bt}$

$\ln\left(\frac{200}{S} - 1\right) = \ln a + (b\ln 2)t$

b) $\ln a = \ln 4$

$a = 4$

$b\ln 2 = \frac{\ln 4 - 0}{0 - 4}$

$b\ln 2 = \frac{\ln 4}{-4}$

$b = -\frac{1}{2}$

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b (ii) $S = \dfrac{200}{1+4(2^{-0.5t})}$

$S = 200\left[1+4(2^{-0.5t})\right]^{-1}$

$\dfrac{dS}{dt} = -200\left[1+4(2^{-0.5t})\right]^{-2}\left[4(2^{-0.5t}\ln 2)(-\frac{1}{2})\right]$

$= \dfrac{400\,(2^{-0.5t}\ln 2)}{\left[1+4(2^{-0.5t})\right]^2}$

$\dfrac{dS}{dt} = 400\ln 2\left[\dfrac{2^{-0.5t}}{\left[1+4(2^{-0.5t})\right]^2}\right]$

$\dfrac{d^2S}{dt^2} = 400\ln 2\left[\dfrac{\left[1+4(2^{-0.5t})\right]^2\left[2^{-0.5t}\ln 2(-\frac{1}{2})\right]-2^{-0.5t}(2)\left[1+4(2^{-0.5t})\right]\left[4(2^{-0.5t}\ln 2(-\frac{1}{2})\right]}{\left[1+4(2^{-0.5t})\right]^4}\right]$

$= 400\ln 2\left[1+4(2^{-0.5t})\right]\left[\dfrac{-2^{-0.5t}\ln 2\left[1+4(2^{-0.5t})\right]+4(2^{-0.5t})(2^{-0.5t}\ln 2)}{2}\right]$

$\left[1+4(2^{-0.5t})\right]^4$

$= \dfrac{400\ln 2\left[-\ln 2(2^{-0.5t})\left[1+4(2^{-0.5t})\right]+4\ln 2(2^{-t})\right]}{\left[1+4(2^{-0.5t})\right]^3}$

b (iii) $\dfrac{dS}{dt} = \dfrac{400\,(2^{-0.5t}\ln 2)}{\left[1+4(2^{-0.5t})\right]^2}$

$> 0 \qquad (\text{for } 0 \le t \le 48)$

$\therefore$ S is increasing in the first 48 hours after the start of the experiment.

$\dfrac{d^2S}{dt^2} = \dfrac{400\ln 2\left[-\ln 2(2^{-0.5t})\left[1+4(2^{-0.5t})\right]+4\ln 2(2^{-t})\right]}{\left[1+4(2^{-0.5t})\right]^3}$

$< 0 \qquad \text{for } (0 \le t \le 48)$

$\therefore$ $\dfrac{dS}{dt}$ will attend to its maximum value during the first 48 hours after the start of the experiment.

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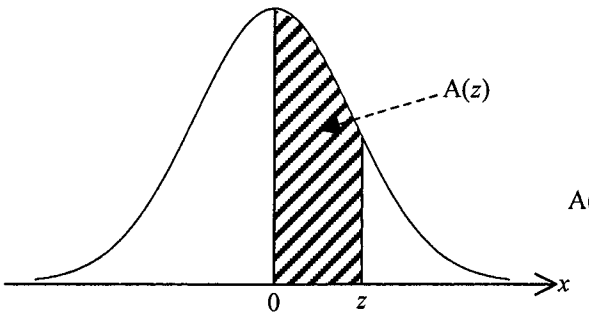
Answers written in the margins will not be marked.

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$

Comments

The candidate demonstrates comprehensive knowledge and understanding of the concepts underpinning calculus and statistics in the curriculum by applying them successfully at a sophisticated level to a wide range of unfamiliar situations, such as in Questions 1, 2, 3, 4, 6, 7, 9 and 11.

He/She communicates and expresses views and arguments precisely and logically using mathematical language and notations. Typical examples are his/her solutions in Questions 1 to 7, 9 and 11.

He/She formulates mathematical models successfully in complex situations, employs appropriate strategies to arrive at a complete solution, and evaluates the significance and reasonableness of the results obtained, as in Questions 1(b), 2(b), 3(b)(ii), 4(c), 6, 7(b), 9(b) and 11.

It can be concluded that the candidate has the ability to integrate knowledge and skills from different areas of the curriculum in handling complex tasks using a variety of strategies.