Level 5 Paper 1
exemplar with comments

2017-DSE
MATH CP

PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2017

Please stick the barcode label here.

Candidate Number									

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

8.30 am – 10.45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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Answers written in the margins will not be marked.

SECTION A(1) (35 marks)

1. Make y the subject of the formula $k = \dfrac{3x - y}{y}$. (3 marks)

$k = \dfrac{3x-y}{y}$

$ky = 3x - y$

$ky + y = 3x$

$y = \dfrac{3x}{k+1}$

2. Simplify $\dfrac{(m^4 n^{-1})^3}{(m^{-2})^5}$ and express your answer with positive indices. (3 marks)

$\dfrac{(m^4 n^{-1})^3}{(m^{-2})^5}$

$= \dfrac{m^{12} n^{-3}}{m^{-10}}$

$= \dfrac{m^{22}}{n^3}$

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3. Factorize

(a) $x^2 - 4xy + 3y^2$,

(b) $x^2 - 4xy + 3y^2 + 11x - 33y$.

(3 marks)

a) $x^2 - 4xy + 3y^2$

$= (x-3y)(x-y)$

b) $x^2 - 4xy + 3y^2 + 11x - 33y$

$= (x-3y)(x-y) + 11(x-3y)$

$= (x-3y)(x-y+11)$

4. There are only two kinds of admission tickets for a theatre: regular tickets and concessionary tickets. The prices of a regular ticket and a concessionary ticket are \$126 and \$78 respectively. On a certain day, the number of regular tickets sold is 5 times the number of concessionary tickets sold and the sum of money for the admission tickets sold is \$50 976 . Find the total number of admission tickets sold that day.

(4 marks)

Let the number of concessionary tickets sold be n, then number of regular tickets sold is $5n$.

$126(5n) + 78n = 50976$

$n = 72$

$\therefore$ Total number of admission tickets sold

$= 6n$

$= 6 \times 72$

$= 432$

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5. (a) Find the range of values of x which satisfy both $7(x-2) \le \frac{11x+8}{3}$ and $6-x<5$.

(b) How many integers satisfy both inequalities in (a)?

(4 marks)

a) $7(x-2) \le \frac{11x+8}{3}$ and $6-x<5$

$21(x-\cancel{4}2) \le 11x+8$ and $-x<-1$

$10x \le 50$ and $x>1$

$x \le 5$ and $x>1$

$\therefore$ The required range of values of x is $1<x\le 5$

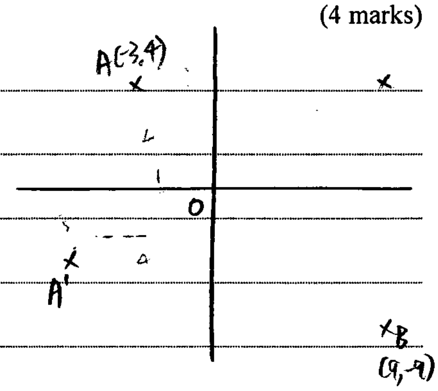
b) 4

6. The coordinates of the points A and B are $(-3,4)$ and $(9,-9)$ respectively. A is rotated anticlockwise about the origin through $90°$ to A' . B' is the reflection image of B with respect to the x-axis.

(a) Write down the coordinates of A' and B' .

(b) Prove that AB is perpendicular to $A'B'$.

(4 marks)



a) Coordinates of A' : $(-4,-3)$

Coordinates of B' : $(9,9)$

b) Slope of $AB = \frac{4-(-9)}{(-3)-9} = -\frac{13}{12}$

Slope of $A'B' = \frac{9-(-3)}{9-(-4)} = \frac{12}{13}$

$\because$ Slope of $AB \times$ Slope of $A'B' = -1$

$\therefore AB \perp A'B'$

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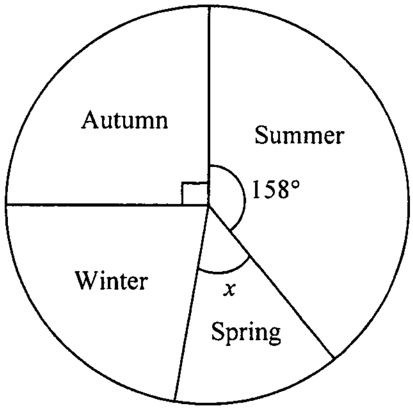
7. The pie chart below shows the distribution of the seasons of birth of the students in a school.



Distribution of the seasons of birth of the students in the school

If a student is randomly selected from the school, then the probability that the selected student was born in spring is $\frac{1}{9}$.

(a) Find x .

(b) In the school, there are 180 students born in winter. Find the number of students in the school.

(4 marks)

a) $\because$ P(selected student is born in spring) $= \frac{1}{9}$

$\therefore$ x should also account for $\frac{1}{9}$ of the graph

$\therefore$ $\frac{x}{360} = \frac{1}{9}$

$x = 40$

$\therefore$ The value of x is $40°$

b) Percentage of students born in winter

$= \frac{360-40-90-158}{360} \times 100\%$

$= 20\%$

$\therefore$ let total number of students be n

$0.2n = 180$

$n = 900$

$\therefore$ Required number of students in the school $= 900$

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8. It is given that y varies inversely as $\sqrt{x}$. When $x = 144$, $y = 81$.

(a) Express y in terms of x .

(b) If the value of x is increased from 144 to 324 , find the change in the value of y .

(5 marks)

a) Let $y = \frac{k}{\sqrt{x}}$, where k is a non-zero constant

$81 = \frac{k}{\sqrt{144}}$

$k = 972$

$\therefore y = \frac{972}{\sqrt{x}}$

b) Let new value of y be y'

$y' = \frac{972}{\sqrt{324}}$

$y' = \frac{972}{18}$

$y' = 54$

$\therefore$ Required change in value of y

$= 54 - 81$

$= -27$

$\therefore$ The value of y is decreased by 27.

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9. A bottle is termed *standard* if its capacity is measured as 200 mL correct to the nearest 10 mL .

(a) Find the least possible capacity of a *standard* bottle.

(b) Someone claims that the total capacity of 120 *standard* bottles can be measured as 23.3 L correct to the nearest 0.1 L . Do you agree? Explain your answer.

(5 marks)

a) Required least possible capacity

$= 200 - \left(\frac{10}{2}\right)$ mL

$= 195$ mL

b) least possible actual total capacity of 120 bottles

$= 195 \times 120$

$= 23.4$ L

> 23.3 L

$\therefore$ The claim is disagreed.

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SECTION A(2) (35 marks)

10. In Figure 1, $OPQR$ is a quadrilateral such that $OP = OQ = OR$. OQ and PR intersect at the point S. S is the mid-point of PR.

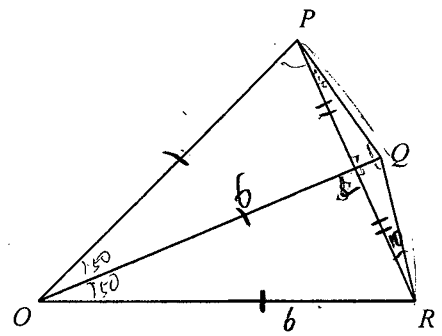


Figure 1

(a) Prove that $\Delta OPS \cong \Delta ORS$. (2 marks)

(b) It is given that O is the centre of the circle which passes through P, Q and R. If $OQ = 6$ cm and $\angle PRQ = 10°$, find the area of the sector $OPQR$ in terms of π. (4 marks)

a) In ΔOPS and ΔORS,

$OP = OR$ (given)

Consider ΔOPR,

$\because OP = OR$

$\therefore \angle OPR = \angle ORP$ (base $\angle$s, isos. Δ)

$PS = RS$ (given)

$\therefore \Delta OPS \cong \Delta ORS$ (SAS)

b) Consider ΔPRQ,

$\because PS = RS$ (given) and $\angle PSQ = \angle RSQ = 90°$ (corr. $\angle$s, $\cong \Delta$s)

$\therefore \angle QPR = \angle PRQ = 10°$ (base $\angle$s, isos. Δ)

$\therefore \angle PQR = 180° - 10° - 10° = 160°$

$\therefore$ reflex $\angle PQR = 360° - 160° = 200°$ ($\angle$s at a pt.)

$\therefore \angle POR = 100°$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\because \angle POQ = \angle ROQ$ (corr. $\angle$s, $\cong \Delta$s)

$\therefore \angle POQ = \angle ROQ = 50°$

$\therefore$ Area of sector $OPQR$

$= \pi(6)^2\left(\frac{100}{360}\right)$

$= 10\pi \text{ cm}^2$

11. The stem-and-leaf diagram below shows the distribution of the hourly wages (in dollars) of the workers in a group.

Stem (tens)	Leaf (units)								
6	1	1	1	3	4	6	8	9	9
7	a	7	7	8					
8	1	b							

It is given that the mean and the range of the above distribution are $70 and $22 respectively.

(a) Find the median and the standard deviation of the above distribution. (5 marks)

(b) If a worker is randomly selected from the group, find the probability that the hourly wage of the selected worker exceeds $70 . (2 marks)

a) $\because$ Range $= 22$

$\therefore (80+b) - 61 = 22$

$b = 3$

$\because$ Mean $= 70$

$\therefore \dfrac{978 + (70+a)}{15} = 70$

$a = 2$

$\therefore$ Median $= 69$

$\therefore$ Standard deviation

$= \sqrt{\dfrac{(61-70)^2 + (61-70)^2 + (61-70)^2 + (63-70)^2 + \dots (83-70)^2}{15}}$

$= 7.33$ (3 s.f.)

b) P(hourly wage of selected worker exceeds $70)

$= \dfrac{6}{15}$

$= 0.4$

Answers written in the margins will not be marked.

12. A solid metal right prism of base area 84 cm^2 and height 20 cm is melted and recast into two similar solid right pyramids. The bases of the two pyramids are squares. The ratio of the base area of the smaller pyramid to the base area of the larger pyramid is $4:9$.

(a) Find the volume of the larger pyramid. (3 marks)

(b) If the height of the larger pyramid is 12 cm, find the total surface area of the smaller pyramid. (4 marks)

a) Original volume of prism

$= \frac{1}{3}(84)(20)$

$= 560\text{ cm}^3$

$\because$ The two new pyramids are similar

$\therefore \left(\frac{\text{Base area of small pyramid}}{\text{Base area of large pyramid}}\right)^3 = \left(\frac{\text{Volume of small pyramid}}{\text{Volume of large pyramid}}\right)^2$

$\therefore$ Volume ratio of small pyramid to large pyramid

$= 8:27$

$\therefore$ Volume of larger pyramid

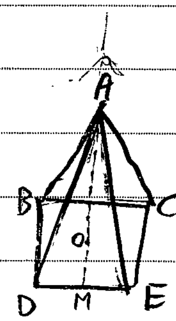
$= 560\left(\frac{27}{8+27}\right)\text{ cm}^3$

$= 432\text{ cm}^3$

b) Base area of larger pyramid

$= 108\text{ cm}^2$

Construct figure as shown



Consider ΔAOM,

$AO^2 + OM^2 = AM^2$ (Pyth. theorem)

$\therefore AM = \sqrt{171}\text{ cm}$

$\therefore$ Total surface area of large pyramid

$= 108 + \frac{1}{2}(\sqrt{171})(\sqrt{108}) \times 4\text{ cm}^2$

$= 379.794\text{ cm}^2$

Again, by similarity, area ratio of small pyramid to large pyramid $= 4:9$

$\therefore$ Total surface area of smaller pyramid $= 379.794 \times \left(\frac{4}{9}\right)$

$= 169\text{ cm}^2$ (3 s.f.)

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13. The coordinates of the points E, F and G are $(-6,5)$, $(-3,11)$ and $(2,-1)$ respectively. The circle C passes through E and the centre of C is G.

(a) Find the equation of C. (2 marks)

(b) Prove that F lies outside C. (2 marks)

(c) Let H be a moving point on C. When H is farthest from F,

(i) describe the geometric relationship between F, G and H;

(ii) find the equation of the straight line which passes through F and H.

(3 marks)

a) $\because$ G is center of C and C passes through E

$\therefore$ EG is a radius of the circle

$\because$ EG = 10 units

$\therefore$ Equation of C

$(x-2)^2+(y+1)^2=100$

b) Length of FG

$=\sqrt{[(-3)-2]^2+[11-(-1)]^2}$

$=13$ units

>10 units (radius of circle)

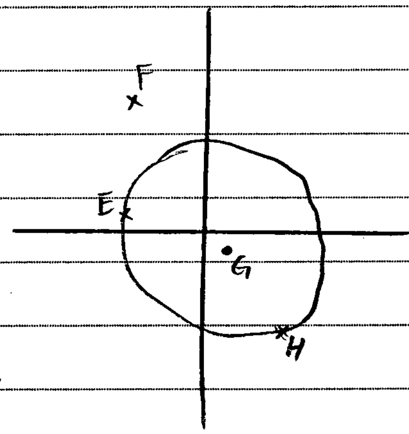
$\therefore$ F lies outside C

c)i) F, G and H are collinear

ii) $\because$ When H is farthest from F,

F, G and H are collinear

$\therefore$ An equation of straight line which passes through F and H also passes through G



$\therefore$ Slope of FG $=\frac{11-(-1)}{(-3)-2}=-2.4$

$\therefore$ Equation of straight line

$(y+1)=-\frac{12}{5}(x-2)$

$-5y-5=12x-24$

$\therefore$ The equation of straight line required : $12x+5y-19=0$

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14. Let $f(x) = 6x^3 - 13x^2 - 46x + 34$. When $f(x)$ is divided by $2x^2 + ax + 4$, the quotient and the remainder are $3x + 7$ and $bx + c$ respectively, where a, b and c are constants.

(a) Find a. (3 marks)

(b) Let $g(x)$ be a quadratic polynomial such that when $g(x)$ is divided by $2x^2 + ax + 4$, the remainder is $bx + c$.

(i) Prove that $f(x) - g(x)$ is divisible by $2x^2 + ax + 4$.

(ii) Someone claims that all the roots of the equation $f(x) - g(x) = 0$ are integers. Do you agree? Explain your answer.

(5 marks)

a) $f(x) = 6x^3 - 13x^2 - 46x + 34$

$f(x) = (2x^2 + ax + 4)(3x+7) + bx + c$

$= 6x^3 + 14x^2 + 3ax^2 + 7ax + 12x + 28 + bx + c$

$= 6x^3 + (3a+14)x^2 + (12 + 7a + b)x + (28 + c)$

By comparing the like terms, we have

$3a + 14 = -13$

$a = -9$

$\therefore$ The value of a is -9

b)i) Let $g(x) = (2x^2 - 9x + 4)Q(x) + bx + c$

$\therefore f(x) - g(x)$

$= (2x^2 - 9x + 4)(3x+7) + bx + c - [(2x^2 - 9x + 4)Q(x) + bx + c]$

$= (2x^2 - 9x + 4)(3x+7) - (2x^2 - 9x + 4)Q(x)$

$= (2x^2 - 9x + 4)[3x + 7 - Q(x)]$

$\therefore$ When $f(x) - g(x)$ is divided by $2x^2 + ax + 4$, the remainder is 0

$\therefore f(x) - g(x)$ is divisible by $2x^2 + ax + 4$

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SECTION B (35 marks)

15. Let a and b be constants. Denote the graph of $y = a + \log_b x$ by G. The x-intercept of G is 9 $(9, 0)$ and G passes through the point $(243, 3)$. Express x in terms of y. (4 marks)

$G: y = a + \log_b x$

$\because$ x-intercept of $G = 9$

$\therefore$ $a + \log_b 9 = 0$ —①

$\because$ G passes through $(243, 3)$

$\therefore$ $3 = a + \log_b 243$ —②

From ① and ②,

$3 = \log_b 243 - \log_b 9$

$3 = \log_b 27$

$b = 3$ —③

Sub ③ into ①,

$a + \log_3 9 = 0$

$a = -2$

$\therefore$ $y = -2 + \log_3 x$

$x = 3^{y+2}$

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16. A city adopts a plan to import water from another city. It is given that the volume of water imported in the 1st year since the start of the plan is $1.5\times10^{7}\ \text{m}^3$ and in subsequent years, the volume of water imported each year is 10% less than the volume of water imported in the previous year.

(a) Find the total volume of water imported in the first 20 years since the start of the plan. (2 marks)

(b) Someone claims that the total volume of water imported since the start of the plan will not exceed $1.6\times10^{8}\ \text{m}^3$. Do you agree? Explain your answer. (2 marks)

a) Total volume of water imported required

$$= \frac{1.5\times10^7[1-(0.9)^{20}]}{1-0.9}$$

$= 1.32\times10^8\ m^3$ (3 s.f.)

b) Total volume of water imported

$$= \frac{1.5\times10^7}{1-0.9}$$

$= 1.5\times10^8\ m^3$

$< 1.6\times10^8\ m^3$

∴ The claim is agreed.

Answers written in the margins will not be marked.

17. In a bag, there are 4 green pens, 7 blue pens and 8 black pens. If 5 pens are randomly drawn from the bag at the same time,

(a) find the probability that exactly 4 green pens are drawn; (2 marks)

(b) find the probability that exactly 3 green pens are drawn; (2 marks)

(c) find the probability that not more than 2 green pens are drawn. (2 marks)

a) P(exactly 4 green pens drawn)

$= \frac{({}_4C_4)({}_{15}C_1)}{({}_{19}C_5)}$

$= \frac{5}{3876}$

$= 0.00129$ (3 s.f.)

b) P(exactly 3 green pens drawn)

$= \frac{({}_4C_3)({}_{15}C_2)}{({}_{19}C_5)}$

$= \frac{35}{969}$

$= 0.0361$ (3 s.f.)

c) P(not more than 2 green pens are drawn)

$= \frac{{}_{15}C_5 + ({}_4C_1)({}_{15}C_4) + ({}_4C_2)({}_{15}C_3)}{{}_{19}C_5}$

$= \frac{3731}{3876}$

$= 0.963$ (3 s.f.)

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18. The equation of the parabola Γ is $y=2x^2-2kx+2x-3k+8$, where k is a real constant. Denote the straight line $y=19$ by L .

(a) Prove that L and Γ intersect at two distinct points. (3 marks)

(b) The points of intersection of L and Γ are A and B .

(i) Let a and b be the x-coordinates of A and B respectively. Prove that $(a-b)^2=k^2+4k+23$.

(ii) Is it possible that the distance between A and B is less than 4 ? Explain your answer. (5 marks)

a) When L and Γ intersect,

$2x^2-2kx+2x-3k+8=19$

$2x^2+(2-2k)x-(11+3k)=0$

$\Delta=(2-2k)^2-4(2)(-11-3k)$

$=4-8k+4k^2+88+24k$

$=4k^2+16k+84$

$=4(k^2+4k)+84$

$=4[(k+2)^2-2^2]+84$

$=4(k+2)^2+68$

$\therefore$ We can see that the minimum value of the discriminant Δ is also greater than 0

$\therefore$ L and Γ intersect at two distinct points

b)i) $(a-b)^2$

$=(a+b)^2-4ab$

$=\left(\frac{2k-2}{2}\right)^2-4\left(\frac{-11-3k}{2}\right)$

$=k^2+4k+23$

ii) $\because$ $(a-b)^2=k^2+4k+23$ (proved)

$\therefore$ $k^2+4k+23$

$=(k+2)^2-2^2+23$

$=(k+2)^2+19$

From this, we can see that the minimum value of $(a-b)^2$ is

Answers written in the margins will not be marked.

19, so the minimum value of $(a-b)$ is 4.36 or -4.36 (rej.)

$\because$ Minimum value of $AB > 4$

$\therefore$ It is not possible that distance between A and B is less than 4.

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19. ABC is a thin triangular metal sheet, where $BC = 24$ cm , $\angle BAC = 30°$ and $\angle ACB = 42°$.

(a) Find the length of AC . (2 marks)

(b) In Figure 2, the thin metal sheet ABC is held such that only the vertex B lies on the horizontal ground. D and E are points lying on the horizontal ground vertically below the vertices A and C respectively. AC produced meets the horizontal ground at the point F . A craftsman finds that $AD = 10$ cm and $CE = 2$ cm .

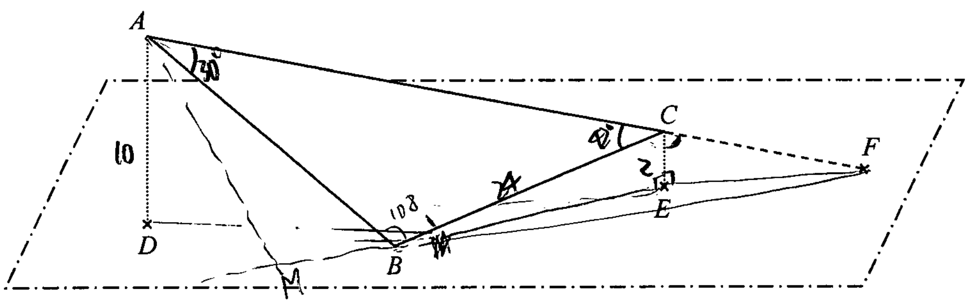

Figure 2

(i) Find the distance between C and F .

(ii) Find the area of ΔABF .

(iii) Find the inclination of the thin metal sheet ABC to the horizontal ground.

(iv) The craftsman claims that the area of ΔBDF is greater than 460 cm^2 . Do you agree? Explain your answer.

(11 marks)

a) Consider ΔABC,

$\frac{24}{\sin 30} = \frac{AB}{\sin 42°}$

$AB = 32.118269$ cm

$\therefore AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$

$AC = 45.650713$ cm

$\therefore$ The length of AC is 45.7 cm (3 s.f.)

b) i) Consider ΔFCE and ΔFAD, they are similar triangles

From a), $AC = 45.650713$ cm

$\frac{CF}{CF + AC} = \frac{CE}{AD}$ (corr. sides, $\sim \Delta$s)

$\therefore 10CF = 2(CF + 45.65)$

$CF = 11.412678$ cm

$\therefore$ The length of CF is 11.4 cm (3 s.f.)

ii) Consider $\triangle ABF$,

$AF = 57.06339$ cm (proved)

$AB = 32.118269$ cm (proved)

$\angle BAC = 30^\circ$ (given)

$\therefore$ Area of $\triangle ABF$

$= \frac{1}{2}(AF)(AB)\sin\angle BAC$

$= 458 \text{ cm}^2$ (3 s.f.)

iii) Reconstruct figure as shown, such that $AM \perp MF$.

Consider $\triangle ABF$,

$$\frac{AB}{\sin\angle AFB} = \frac{AF}{\sin\angle ABF}$$

$\angle AFB = 28.76964^\circ$

Consider $\triangle AMF$,

$\sin\angle AFM = \frac{AM}{AF}$

$AM = 27.463997$ cm

Consider $\triangle ADM$,

$\sin\angle AMD = \frac{AD}{AM}$

$\angle AMD = 21.353^\circ$

$\therefore$ The inclination of thin metal sheet ABC is 21.4° (3 s.f.)

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END OF PAPER

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Comments

The candidate has an excellent mastery of algebraic manipulation skills, which enables him/her to solve the questions in Section A accurately and precisely. He/She is able to solve questions in statistics by applying relevant formulas. He/She is also able to solve questions involving geometric figures proficiently by using concepts in deductive geometry, coordinate geometry, mensuration and trigonometry. This demonstrates that the candidate has a comprehensive knowledge and understanding of the mathematical concepts in all three strands of the curriculum.

In addition, the candidate is capable of presenting proofs and solutions for the questions logically and precisely, using relevant symbols and mathematical language, including equations and inequalities, to express his/her views and ideas.

His/Her performance in Questions 9, 13, 18 and 19 demonstrates that the candidate recognizes the meaning and significance of the results obtained in the first few parts of the questions. The candidate makes further deductions and thus comes to the correct conclusions. This demonstrates that the candidate has the ability to explore the relationship between different parts of the harder questions and to draw conclusions through logical deductions.

It can be concluded that the candidate demonstrates comprehensive knowledge and understanding of the mathematical concepts in the Compulsory Part and is capable of expressing views precisely and logically using mathematical language and notations. Also, the candidate has the ability to apply and integrate knowledge and skills from different areas of the Compulsory Part to handle complex tasks.