Level 5 Module 1
exemplar with comments

**2017-DSE
MATH EP
M1**

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2017

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of TWO sections, A and B.
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.
7. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Please stick the barcode label here.

Candidate Number								

A031E001

Answers written in the margins will not be marked.

SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where k is a constant:

x	0	2	4	5	8	9
$P(X=x)$	k^2	0.16	0.18	0.3	k	0.12

Find

(a) k,

(b) $E(X)$,

(c) $Var(2-3X)$.

(6 marks)

(a) $k^2 + 0.16 + 0.18 + 0.3 + k + 0.12 = 1$

$k^2 + k = 0.24$

$k = 0.2$ or $k = -1.2$ (rejected)

$\therefore k = 0.2$

(b) $E(X) = 2 \times 0.16 + 4 \times 0.18 + 5 \times 0.3 + 8 \times 0.2 + 9 \times 0.12$

$E(X) = 5.22$

(c) $Var(2-3X)$

$= 9 Var(X)$

$= 9(E(X^2) - E(X)^2)$

$= 9(33.54 - 5.22^2)$

$= 56.6244$ //

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

2. Let A and B be two events. Suppose that $P(A)=0.2$, $P(B')=0.7$ and $P(A|B)=0.6$, where B' is the complementary event of B .

(a) Find $P(B|A)$.

(b) Are A and B mutually exclusive? Explain your answer.

(c) Are A and B independent? Explain your answer.

(6 marks)

(a) $P(B) = 0.3$

$0.6 = \frac{P(A\cap B)}{0.3}$

$P(A\cap B) = 0.18$

$P(B|A) = \frac{0.18}{0.2}$

$= 0.9$

(b) $P(A\cap B) = 0.18$

$\neq 0$

$\therefore$ A and B are not mutually exclusive.

(c) $P(A) \times P(B) = 0.06$

$\neq$ ~~0.18~~ $P(A\cap B)$)

$\therefore$ A and B are not independent.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

3. In a large farm, the weights of chickens follow a normal distribution with a mean of μ kg and a standard deviation of σ kg . It is given that the percentage of chickens being lighter than 1.83 kg is the same as the percentage of those being heavier than 3.43 kg . Moreover, 89.04% chickens weigh between 1.83 kg and 3.43 kg .

(a) Find μ and σ .

(b) If 9 chickens are selected randomly from the farm, find the probability that the mean of their weights lies between 2.5 kg and 3.1 kg .

(5 marks)

(a) $\cancel{P(1.83\text{ kg} \le X \le 3.43\text{ kg})}$

$\cancel{= P\left(\frac{1.83-\mu}{6} \le Z \le \frac{3.43-\mu}{6}\right)}$

$\cancel{= 0.8904}$

$\cancel{A(1.83}$

$P(X < 1.83) = P(X > 3.43)$

$P\left(Z < \frac{1.83-\mu}{6}\right) = P\left(X > \frac{3.43-\mu}{6}\right)$

$-\left(\frac{1.83-\mu}{6}\right) = \frac{3.43-\mu}{6}$

$-1.83+\mu = 3.43-\mu$

$\mu = 2.63$

$P(1.83\text{ kg} \le X \le 3.43\text{ kg}) = 0.8904$

$P\left(\frac{-0.8}{6} \le Z \le \frac{0.8}{6}\right) = 0.8904$

$A\left(\frac{0.8}{6}\right) = 0.4452$

$6 = 0.5$

(b) $P(2.5\text{kg} \le \bar{X} \le 3.1\text{kg})$

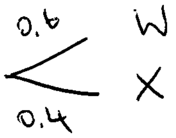
$= P\left(\frac{2.5-2.63}{\frac{0.5}{3}} \le Z \le \frac{3.1-2.63}{\frac{0.5}{3}}\right)$

$= \cancel{0.3264+} P(-0.78 \le Z \le 2.82)$

$= 0.2823 + 0.4976$

$= 0.7799$

Answers written in the margins will not be marked.



4. Susan plays a game. In each trial of the game, her probability of winning a doll is 0.6 . Susan plays the game until she wins a doll.

(a) Find the probability that Susan wins a doll at the 4th trial in the game.

(b) If Susan cannot win a doll in k trials, then the probability that she wins a doll within 10 trials in the game is greater than 0.95 . Find the greatest value of k .

(c) In each trial of the game, Susan has to pay \$15 . Find the expected amount of money she has to pay to win a doll in the game.

(7 marks)

(a) Probability $= (0.4)^3(0.6)$

$= 0.0384$

(b) $\dfrac{(0.4)^{k-1}(0.6)}{(0.4)^k} > 0.95$

$0.4^k \times \dfrac{1}{0.4} > \cancel{0.95 \times 0.4^k}\ \dfrac{19}{12} \times 0.4^k$

$0.4^k > \dfrac{19}{30} \times 0.4^k$

(c) ~~Proba~~ Expected number of winning a doll $= \dfrac{1}{0.6}$

$= \dfrac{5}{3}$

Expected amount of money $= \dfrac{5}{3} \times 15$

$= \$25$

5. (a) Expand $(1+e^{3x})^2$ in ascending powers of x as far as the term in x^2.

(b) Find the coefficient of x^2 in the expansion of $(5-x)^4(1+e^{3x})^2$.

(6 marks)

$(1+e^{3x})^2$

(a) $= 1 + 2e^{3x} + e^{6x}$

$e^{3x} = 1 + 3x + \frac{9}{2}x^2 + \dots\dots$

$e^{6x} = 1 + 6x + 18x^2 + \dots\dots$

$(1+e^{3x})^2$

$= 1 + 2(1 + 3x + \frac{9}{2}x^2) + 1 + 6x + 18x^2 + \dots\dots$

$= 1 + 2 + 6x + 9x^2 + 1 + 6x + 18x^2 + \dots\dots$

$= 4 + 12x + 27x^2 + \dots\dots$

(b) $(5-x)^4 = 625 - 4(5)^3(x) + 6(5)^2(x^2) + \dots\dots$

$= 625 - 500x + 150x^2 + \dots\dots$

$(5-4)^x(1+e^{3x})^2$

Coefficient of x^2

$= 625(27) - 500(12) + 150(4)$

$= 11475$ //

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

6. Let $f(x) = 4x^3 + mx^2 + nx + 615$, where m and n are constants. It is given that $(6, -33)$ is a turning point of the graph of $y = f(x)$. Find

(a) m and n ,

(b) the minimum value(s) and the maximum value(s) of $f(x)$.

(6 marks)

(a) $-33 = 4(6)^3 + m(6)^2 + 6n + 615$

$-1512 = 36m + 6n \quad \cdots (i)$

$f'(x) = 12x^2 + 2mx + n$

$0 = 12(6)^2 + 2m(6) + n$

$-432 = 12m + n \quad \cdots (ii)$

$m = -30 \; , \; n = -72$

(b) $F(x) = 4x^3 - 30x^2 - 72x + 615$

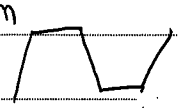
$f'(x) = 12x^2 - 60x - 72$

when $f'(x) = 0$, $12x^2 - 60x - 72 = 0$

$x = 6$ or $x = -1$

	$x<-1$	$x=-1$	$-1<x<6$	$x=6$	$x>6$
$f'(x)$	$+$	0	$-$	0	$+$

When $x = -1$, $f(x)$ attains maximum



Maximum value $= 653$

When $x = 6$, $f(x)$ attains minimum

Minimum value $= -33$ //

Answers written in the margins will not be marked.

7. Consider the curve $C : y = \dfrac{x}{\sqrt{x-2}}$, where $x > 2$.

(a) Find $\dfrac{dy}{dx}$.

(b) A tangent to C passes through the point $(9, 0)$. Find the slope of this tangent.

(7 marks)

(a) $y = \dfrac{x}{\sqrt{x-2}}$

$\dfrac{dy}{dx} = \dfrac{(\sqrt{x-2})(1) - (x)(\frac{1}{2})(x-2)^{-\frac{1}{2}}}{(x-2)}$

$= \dfrac{\sqrt{x-2} - \frac{x}{2\sqrt{x-2}}}{(x-2)}$

$= \dfrac{\frac{1}{\sqrt{x-2}}\left(x-2-\frac{x}{2}\right)}{(x-2)}$

$= \dfrac{\frac{1}{2}x - 2}{(x-2)^{\frac{3}{2}}}$

$= \dfrac{(x-4)}{2(x-2)^{\frac{3}{2}}}$

~~(b)~~ $\cancel{\left.\dfrac{dy}{dx}\right|_{x=9} = \dfrac{(9-4)}{2(9-2)^{\frac{3}{2}}}}$

$\cancel{= 0.1350 \text{ (cor to 4 dp)}}$

$\dfrac{y}{x-9} = \dfrac{(x-4)}{2(x-2)^{\frac{3}{2}}}$

$\dfrac{\frac{x}{\sqrt{x-2}}}{x-9} = \dfrac{(x-4)}{2(x-2)^{\frac{3}{2}}}$

$\dfrac{x(2)(x-2)^{\frac{3}{2}}}{(\sqrt{x-2})(x-9)}$

$\dfrac{(2x)(x-2)}{(x-9)} = x-4$

$2x^2 - 4x = x^2 - 13x + 36$

$x = -12$ or $x = 3$

(rejected)

(when $x=3$, $y=3$)

Slope of tangent $= \dfrac{9-3}{0-3}$

$= -2$ //

Answers written in the margins will not be marked.

$x(\sqrt{x-2})^{-1}$

$= (\sqrt{x-2})^{-1} + x(-1)(\sqrt{x-2})^{-2}$

$= \frac{1}{\sqrt{x-2}} - \frac{x}{(x-2)}$

$= \frac{(x-2) - x\sqrt{x-2}}{(x-2)^{\frac{3}{2}}}$

$=$

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

8. Define $g(x)=\frac{1}{x}\ln\left(\frac{e}{x}\right)$ for all $x>0$.

(a) Using integration by substitution, find $\int g(x)\,dx$.

(b) Denote the curve $y=g(x)$ by Γ .

(i) Write down the x-intercept(s) of Γ .

(ii) Find the area of the region bounded by Γ , the x-axis and the straight lines $x=1$ and $x=e^2$.

(7 marks)

(a) ~~$\int g(x) = \int \frac{1}{x}\ln\left(\frac{e}{x}\right) dx$~~

$g(x) = \frac{1}{x}(\ln e - \ln x)$ Let $u = \ln x$

$g(x) = \frac{1}{x}(1-\ln x)$ $\frac{du}{dx} = \frac{1}{x}$

$g(x) = \frac{1}{x} - \frac{\ln x}{x}$ $x\,du = dx$

$\int g(x) = \int \frac{1}{x}\,dx - \int \frac{\ln x}{x}\,dx$

$= [\ln x] - \int u\,du$

$= [\ln x] - \left[\frac{u^2}{2}\right] + C$

$= \ln x - \frac{(\ln x)^2}{2} + C$

(b) (i) $y = g(x)$

$0 = \frac{1}{x}\ln\left(\frac{e}{x}\right)$

$0 = \frac{1}{x}(1-\ln x)$

$0 = \frac{1}{x} - \frac{\ln x}{x}$

$0 = \frac{1}{x}(1-\ln x)$

$\frac{1}{x} = 0$ (rejected) $\ln x = 1$

$x = e$

$\therefore$ The x-intercept is e.

(ii) Area $= \int_1^{e^2} \left(\frac{1}{x}\ln\left(\frac{e}{x}\right)\right) dx$

$= \int_1^{e^2} \left(\frac{1}{x} - \frac{\ln x}{x}\right) dx$

$= \left[\ln x - \frac{(\ln x)^2}{2}\right]_1^{e^2}$

$= 2 - 2 - 0 - 0$

$= 0$ //

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

SECTION B (50 marks)

9. The daily times spent on homework of the students in a school follow a normal distribution with a mean of μ hours and a standard deviation of 0.4 hour.

(a) A survey is conducted in the school to estimate μ .

(i) A sample of 40 students in the school is randomly selected and their daily times spent on homework are recorded below:

Daily time spent (x hours)	Number of students
$0.5 < x \leq 1.0$ 0.75	11
$1.0 < x \leq 1.5$ 1.25	13
$1.5 < x \leq 2.0$ 1.75	8
$2.0 < x \leq 2.5$ 2.25	5
$2.5 < x \leq 3.0$ 2.75	3

Find a 90% confidence interval for μ .

(ii) Find the least sample size to be taken such that the width of a 97% confidence interval for μ is at most 0.3 .

(7 marks)

(b) Suppose that $\mu = 1.48$. If the daily time spent on homework of a student exceeds 2 hours, then the student has to attend homework guidance class.

(i) If a student is randomly selected from the school, find the probability that the student has to attend homework guidance class.

(ii) A sample of 15 students is now randomly drawn from the school and their daily times spent on homework are examined one by one. Given that more than 1 student in the sample have to attend homework guidance class, find the probability that the 10th student is the 2nd student who has to attend homework guidance class.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

(a) (i) $\mu = \frac{11 \times 0.75 + 13 \times 1.25 + 8 \times 1.75 + 5 \times 2.25 + (3 \times 2.75)}{40}$

$= 1.45$

Confidence interval for μ

$= \left(1.45 - 1.645 \times \frac{0.4}{\sqrt{40}}, 1.45 + 1.645 \times \frac{0.4}{\sqrt{40}}\right)$

$= (1.3460, 1.5540)$ (cor to 4dp)

(ii) $2 \times 2.17 \times \frac{0.4}{\sqrt{n}} \leq 0.3$

$0.4 \leq \frac{15}{217}(\sqrt{n})$

$n \geq 33.4855$

(cor to 4dp)

$\therefore$ The least sample size is 34.

(b) (i) $P(X > 2)$

$= P(Z > 1.3)$

$= 0.5 - A(1.3)$

$= 0.5 - 0.4032$

$= 0.0968$

$\therefore$ The probability is 0.0968.

(ii) Probability $= \frac{C_1^9 (0.9032)^8 (0.0968) \times 0.0968}{1 - (0.9032)^{15} - C_1^{15} (0.9032)^{14} (0.0968)}$

$= 0.0861$ //

Answers written in the margins will not be marked.

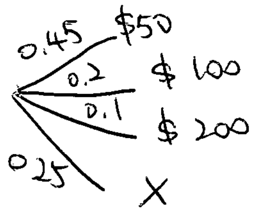
Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

0.947347

10. A department store issues a cash coupon to a customer spending at least $500 in a transaction. The details are given in the following table:

Transaction amount ($\$x$)	Cash coupon
$500 \leq x < 1\,000$	\$50
$1\,000 \leq x < 2\,000$	\$100
$x \geq 2\,000$	\$200



At the department store, 45% , 20% and 10% of the customers each gets one cash coupon of \$50 , \$100 and \$200 respectively in a transaction. Assume that the number of transactions per minute follows a Poisson distribution with a mean of 2 .

(a) Find the probability that there are at most 4 transactions at the department store in a certain minute. (3 marks)

(b) Find the probability that there are exactly 3 transactions at the department store in a certain minute and cash coupons of total value \$200 are issued. (3 marks)

(c) If there are exactly 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value \$200 are issued by the department store in this minute. (3 marks)

(d) Given that there are at most 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value \$200 are issued by the department store in this minute. (3 marks)

10(a) Probability $= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= e^{-2}(2)^0 + e^{-2}(2) + \frac{e^{-2}(2)^2}{2} + \frac{e^{-2}(2)^3}{6} + \frac{e^{-2}(2)^4}{4!}$

$= 0.9473$ (cor to 4 dp)

(b) Probability $= \frac{e^{-2}(2)^3}{3}\left(0.1 \times 0.25^2 \times 3 + (0.2)^2(0.25)(3) + (0.45)^2(0.2)(3)\right)$

$= 0.0307$

(c) Probability $= (0.45)^4 + 4 \times (0.1)(0.25)^3 + 12 \times (0.45)^2(0.25)(0.2) + {}^4C_2 \times (0.2)^2(0.25)^2$

$= 0.1838$ (cor to 4 dp)

(d) Probability $= \dfrac{0.1838(e^{-2}2^4 \div 4!) + 0.0307 + e^{-2}2 \times 0.1 + \frac{e^{-2}2^2}{2!}\left((0.2)^2 + 2 \times 0.1 \times 0.25\right)}{0.947347}$

Answers written in the margins will not be marked.

200 000 × 4
~~5~~50 50 50 50
100 100 0 0
50 100 50 0 12

200 00 × 3
100 100 0 × 3
~~50~~ 50 50 100 × 3

10 (d) probability $= \dfrac{0.016579 + 0.030721 + 0.027067 + 0.0436}{0.947347}$

$= 0.1042$ // (cor to 4 dp)

100 100
200 0 ×2

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

0.74735967

11. Let $f(x)=\dfrac{e^{0.1x}}{x}$. Define $I=\int_{0.5}^{1} f(x)\,dx$. In order to estimate the value of I, Ada suggests using trapezoidal rule with 5 sub-intervals while Billy suggests replacing $e^{0.1x}$ with $1+0.1x+0.005x^2$ and then evaluating the integral.

(a) Find the estimates of I according to the suggestions of Ada and Billy respectively. (5 marks)

(b) Determine each of the two estimates in (a) is an over-estimate or an under-estimate. Explain your answer. (6 marks)

(c) Someone claims that the difference of I and 0.746 is less than 0.002. Do you agree? Explain your answer. (2 marks)

(a) Ada =

$I = \int_{0.5}^{1} \frac{e^{0.1x}}{x}\,dx$

$\Delta x = 0.1$

$I = \frac{0.1}{2}\left(f(0.5)+2f(0.6)+2f(0.7)+2f(0.8)+2f(0.9)+f(1)\right)$

$= \frac{0.1}{2}\left(\frac{e^{0.05}}{0.5}+2\times\frac{e^{0.06}}{0.6}+2\times\frac{e^{0.07}}{0.7}+2\times\frac{e^{0.08}}{0.8}+2\times\frac{e^{0.09}}{0.9}+e^{0.1}\right)$

$= \frac{0.1}{2}(14.95119343)$

$= 0.7476$ (cor to 4 dp)

Billy =

$I = \int_{0.5}^{1} \frac{e^{0.1x}}{x}\,dx$

$= \int_{0.5}^{1} \frac{1+0.1x+0.005x^2}{x}\,dx$

$= \int_{0.5}^{1} \frac{1}{x}\,dx + \int_{0.5}^{1}(0.1)\,dx + \int_{0.5}^{1}(0.005x)\,dx$

$= [\ln x]_{0.5}^{1} + [0.1x]_{0.5}^{1} + \left[\frac{0.005}{2}x^2\right]_{0.5}^{1}$

$=$ ~~0.79002~~ 0.7450 (cor to 4 dp)

(b) $f(x) = \frac{e^{0.1x}}{x}$

$f(x) = e^{0.1x}(x^{-1})$

$f'(x) = e^{0.1x}(0.1)(x^{-1}) + e^{0.1x}(-1)(x^{-2})$

$= \frac{0.1e^{0.1x}}{x} - \frac{e^{0.1x}}{x^2}$

$f'(x) = 0.1e^{0.1x}(x^{-1}) - e^{0.1x}(x^{-2})$

$f''(x) = (0.1e^{0.1x}(0.1)(x^{-1}) - (x^{-2})(0.1)(e^{0.1x})$

$- (0.1e^{0.1x}(x^{-2}) - 2(x^{-3})(e^{0.1x}))$

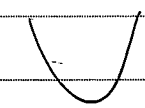
Answers written in the margins will not be marked.

(b) $f''(x) = \left(\frac{0.01e^{0.1x}}{x} - \frac{0.1e^{0.1x}}{x^2}\right) - \frac{0.1e^{0.1x}}{x^2} + \frac{2e^{0.1x}}{x^3}$

$= \frac{0.01e^{0.1x}}{x} - \frac{0.2e^{0.1x}}{x^2} + \frac{2e^{0.1x}}{x^3}$

When $0.5 \le x \le 1$,

$\frac{2e^{0.1x}}{x^3}$ must $> \frac{0.2e^{0.1x}}{x^2}$



∴ $f''(x) > 0$

∴ Ada's estimate is over-estimate.

$e^{0.1x} - 1 - 0.1x - 0.005x^2$

When $0.5 \le x \le 1$,

$e^{0.1x} - 1 - 0.1x - 0.005x^2 > 0$

∴ Billy's estimate is under-estimate.

~~(c)~~

(c) $0.746 - \cancel{0.74}\ 0.74502218$

$= \cancel{0.0009}\ 0.0010$ (cor to 4 dp)

< 0.002

∵ 0.74502218 is under-estimate,

The exact value of I ~~and~~ and 0.746 (difference of) must be less than 0.002.

∴ The ~~cla~~ claim is agreed.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

12. A researcher, Peter, models the number of crocodiles in a lake by

$$x = 4 + \frac{3k}{2^{\lambda t} - k} ,$$

where λ and k are positive constants, x is the number in thousands of crocodiles in the lake and t (≥ 0) is the number of years elapsed since the start of the research.

(a) (i) Express $(x-4)(x-1)$ in terms of λ, k and t.

(ii) Peter claims that the number of crocodiles in the lake does not lie between 1 thousand and 4 thousand. Is the claim correct? Explain your answer.

(3 marks)

(b) Peter finds that $\frac{dx}{dt} = \frac{-\ln 2}{24}(x-4)(x-1)$.

(i) Prove that $\lambda = \frac{1}{8}$.

(ii) For each of the following conditions (1) and (2), find k . Also determine whether the crocodiles in the lake will eventually become extinct or not. If your answer is 'yes', find the time it will take for the crocodiles to become extinct; if your answer is 'no', estimate the number of crocodiles in the lake after a very long time.

(1) When $t = 0$, $x = 0.8$.

(2) When $t = 0$, $x = 7$.

(9 marks)

(a) (i) $\cancel{x = 4 + \frac{3k}{2^{\lambda t}-k}}$

$\cancel{(x-4)(2^{\lambda t}-k) = 3k}$

$(x-4)(x-1) = \left(\frac{3k}{2^{\lambda t}-k}\right)\left(3+\frac{3k}{2^{\lambda t}-k}\right)$

$= 3\left(\frac{3k}{2^{\lambda t}-k}\right) + \left(\frac{3k}{2^{\lambda t}-k}\right)^2$

$= \frac{9k}{2^{\lambda t}-k} + \left(\frac{3k}{2^{\lambda t}-k}\right)^2$

$= \frac{9k(2^{\lambda t}) - 9k^2 + 9k^2}{(2^{\lambda t}-k)^2}$

$= \frac{9k(2^{\lambda t})}{(2^{\lambda t}-k)^2}$ //

Answers written in the margins will not be marked.

(ii) $\cancel{\frac{dx}{dt}}$ ~~When~~ $\cancel{x =}$

$$(x-4)(x-1) = \frac{9k(2^{\lambda t})}{(2^{\lambda t}-k)^2}$$

When x lies between 1 thousand to 4 thousand, ~~the~~ t ~~with~~ will be <0.

$\therefore$ It cannot lie between 1 thousand and 4 thousand,

$\therefore$ The claim is agreed.

(b)(i) $x = 4 + 3k(2^{\lambda t}-k)^{-1}$

$$\frac{dx}{dt} = -3k(2^{\lambda t}-k)^{-2}(2^{\lambda t}\ln 2)(\lambda)$$

$$\frac{-3k(2^{\lambda t})(\ln 2)(\lambda)}{(2^{\lambda t}-k)^2} = \frac{-\ln 2}{24}\left(\frac{9k(2^{\lambda t})}{(2^{\lambda t}-k)^2}\right)$$

$$-72k(2^{\lambda t})(\lambda) = -9k(2^{\lambda t})$$

$$-72(2^{\lambda t})(\lambda) = -9(2^{\lambda t})$$

$$-9 2^{\lambda t}(8\lambda - 1) = 0$$

$$-9 2^{\lambda t} = 0 \text{ (rejected)}$$

or $8\lambda = 1$

$$\lambda = \frac{1}{8} \quad //$$

(ii)(1) When $t = 0$, $x = 0.8$

$$0.8 = 4 + \frac{3k}{2^{(\frac{1}{8})(0)}-k}$$

$$0.8 = 4 + \frac{3k}{1-k}$$

$$-3.2(1-k) = 3k$$

$$-3.2 + 3.2k = 3k$$

$$k = 16$$

$$\frac{dx}{dt} = \frac{-\ln 2}{24}\left(\frac{144(2^{\lambda t})}{(2^{\lambda t}-16)^2}\right)$$

$\because (2^{\lambda t}-16)^2$ must > 0

$\therefore \frac{dx}{dt} < 0$

$\therefore$ The crocodiles will become extinct.

Answers written in the margins will not be marked.

(b)(ii) (1) $x = \cancel{4} 64 + \frac{48}{2^{\frac{1}{8}t}-16}$

$64 + \frac{48}{2^{\frac{1}{8}t}-16} < 0$

$48 < -64(2^{\frac{1}{8}t}-16)$

$-976 < 2^{\frac{1}{8}t}(-64)$

$\therefore 2^{\frac{1}{8}t} < 15.25$

$t > 10.8983$ (cor. to 4dp)

$\therefore$ It will take ~~80~~ 11 years.

(2) When $t=0$, $x=7$,

$3 = \frac{3k}{1-k}$

$3-3k = 3k$

$3 = 6k$

$k = \frac{1}{2}$

$\frac{dx}{dt} = -\frac{\ln 2}{24}\left(\frac{9(\frac{1}{2})(2^{\frac{1}{8}(t)})}{(2^{\frac{1}{8}t}-0.5)^2}\right)$

~~$\because (2^{\frac{1}{8}t}-0.5)^2$ must~~

$\therefore \frac{dx}{dt} > 0$ $\therefore$ It ~~won't~~ won't be extinct.

~~$4+\frac{1.5}{2^{\frac{1}{8}t}-0.5} < 0$~~

~~$1.5 < -4(2^{\frac{1}{8}t}-0.5)$~~

~~$-0.5 < -4(2^{\frac{1}{8}t})$~~

~~$4(2)$~~

~~$2^{\frac{1}{8}t} < 0.125$~~

$\lim_{t\to\infty} x = 4 + \frac{3(-\frac{1}{2})}{(-\frac{1}{2})}$

$= 7$

The number is 7 thousand.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

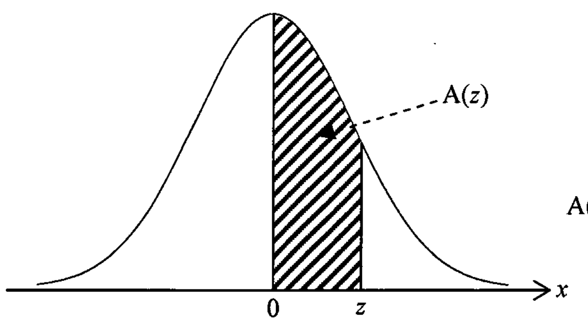
END OF PAPER

Answers written in the margins will not be marked.

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$

Comments

The candidate demonstrates comprehensive knowledge and understanding of the concepts underpinning calculus and statistics in the curriculum by applying them successfully at a sophisticated level to a wide range of unfamiliar situations, such as in Questions 3, 6, 9 and 10.

He/She is able to communicate and express views and arguments precisely and logically using mathematical language and notations. Typical examples are his/her solutions in Questions 2, 3, 5, 6, 9 and 10.

He/She is also able to formulate mathematical models successfully in complex situations, employ appropriate strategies to arrive at a complete solution, and evaluate the significance and reasonableness of the results obtained, such as in Questions 2, 3, 6, 9 and 10.

It can be concluded that the candidate has the ability to integrate knowledge and skills from different areas of the curriculum in handling complex tasks using a variety of strategies.